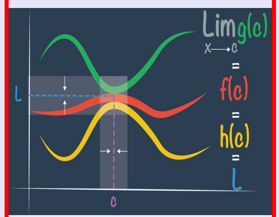
Calculus I

Lecture 33



Feb 19-8:47 AM

Given $f(x) = x^3 - 12x^2 + 36x$

1) Polynomial function $\rightarrow$ Cont. & diff. everywhere.

2) Domain $(-\infty, \infty)$

3) $f(x) = x(x^2 - 12x + 36) = x(x-6)^2$

4) Y-Int $\rightarrow x = 0 \rightarrow y = 0 \rightarrow (0,0)$

5) X-Int. $\rightarrow y = 0 \rightarrow f(x) = 0 \rightarrow x(x-6)^2 = 0 \rightarrow x = 0, x = 6$ Twice

$\rightarrow (0,0), (6,0)$

Twice $\rightarrow$ even # of times

6) $f'(x) = 3x^2 - 24x + 36$

$f'(x) = 0 \rightarrow 3(x^2 - 8x + 12) = 0 \rightarrow 3(x-6)(x-2) = 0$

C.N. $\rightarrow$ 2, 6

C.P. $(2, f(2))$

$(6, f(6))$

7) $f''(x) = 6x - 24$

$f''(x) = 0 \rightarrow 6(x-4) = 0 \rightarrow$ P.I.P. $\rightarrow x = 4$

$\rightarrow (4, f(4))$

8) Sign chart

x	$-\infty$ 2	4	6	∞
$f'(x)$	+	−	−	+
$f''(x)$	−	−	+	+
$f(x)$	Inc., C.D.	Dec., C.D.	Dec., C.U.	Inc., C.U.

Inflection Point $\rightarrow (4, f(4))$

Apr 11-8:46 AM

$f(x) = \dfrac{x^2 - x}{x^2 - 3x + 2}$

1) Domain $\to x^2 - 3x + 2 \neq 0$, $(x-2)(x-1) \neq 0$

$x \neq 2$, $x \neq 1$

$(-\infty, 1) \cup (1, 2) \cup (2, \infty)$

we have a hole at $x=1$

2) Vertical Asy. at $\boxed{x=1}$ $x=2$

3) $\lim\limits_{x \to \infty} f(x) = 1$, $\lim\limits_{x \to -\infty} f(x) = 1 \Rightarrow$ Horizontal Asy. $\to y = 1$

4) $f(x) = \dfrac{x(x-1)}{(x-1)(x-2)} \to f(x) = \dfrac{x}{x-2}$ $\boxed{x \neq 1}$ $x \neq 2$

$f(1) = \frac{1}{1-2} = -1$

5) $f'(x) = \dfrac{1(x-2) - x \cdot 1}{(x-2)^2}$ $f'(x) = \dfrac{-2}{(x-2)^2}$

$f'(x) = -2(x-2)^{-2}$

$f'(x) \neq 0$

$f'(x)$ und. at $x=2$

$\boxed{f'(x) < 0}$

$f(x)$ is decreasing

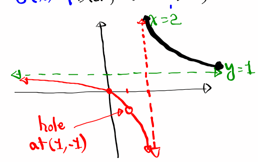
6) $f''(x) = -2 \cdot (-2) \cdot (x-2)^{-2-1} \cdot 1$

$f''(x) = \dfrac{4}{(x-2)^3}$ $f''(x) \neq 0$

$f''(x)$ und. at $x=2$

7) Sign chart

x	$-\infty$	1	2	∞
$f'(x)$	—	—	—	
$f''(x)$	—	—	0 +	
$f(x)$	Dec., CD	Dec., CD	Dec., CU	



use technology and graph $f(x) = \dfrac{x^2 - x}{x^2 - 3x + 2}$, also $f(x) = \dfrac{x}{x-2}$

Apr 11-9:02 AM

$f(x) = \sqrt{x} - \frac{1}{3}x$, $[0, 9]$

$\boxed{f(x) \text{ is cont. on } [0,9]}$

$\boxed{f(x) \text{ is diff. on } (0,9) ?}$ ✓

$f(0) = 0$

$f(9) = \sqrt{9} - \frac{1}{3}(9) = 3 - 3 = 0$

$\boxed{f(0) = f(9)}$

All $\boxed{\text{three conditions}}$ of Rolle's theorem are met, so there is at least one number c in $(0,9)$ such that $f'(c) = 0$

$f(x) = \sqrt{x} - \frac{1}{3}x \to f(x) = x^{1/2} - \frac{1}{3}x$

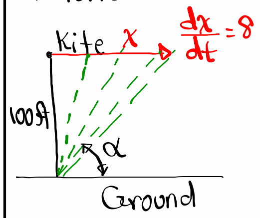
$f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{3}$ $f'(x) = \dfrac{1}{2\sqrt{x}} - \dfrac{1}{3}$ $\to$ Defined on $(0, \infty)$

$f'(c) = 0 \to \dfrac{1}{2\sqrt{c}} - \dfrac{1}{3} = 0 \to \dfrac{1}{2\sqrt{c}} = \dfrac{1}{3}$ $(0,9)$

$2\sqrt{c} = 3 \to \sqrt{c} = \frac{3}{2} \to c = \left(\frac{3}{2}\right)^2$ $c = \frac{9}{4} \to \boxed{c = 2.25}$

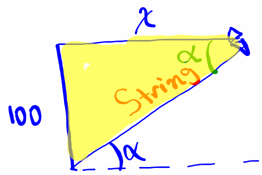
Apr 11-9:21 AM

A Kite is 100 ft above the ground.



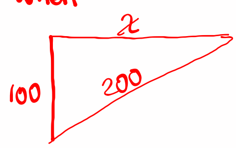
The Kite moves horizontally at the speed of 8ft/s.

At what rate is the angle between the string and ground changing $\frac{d\alpha}{dt}=?$ when 200 ft of string has been let out? $S=200\text{ft}$



$\sin\alpha=\frac{100}{S}$, $\cos\alpha=\frac{x}{S}$, $\tan\alpha=\frac{100}{x}$

when $S=200$



$\sin\alpha=\frac{100}{200}$

$\sin\alpha=\frac{1}{2}$

$\alpha=30°$

$\frac{d}{dt}[\tan\alpha]=\frac{d}{dt}\left[\frac{100}{x}\right]$

$\sec^2\alpha\cdot\frac{d\alpha}{dt}=\frac{-100}{x^2}\cdot\frac{dx}{dt}$

$100^2+x^2=200^2 \rightarrow x^2=200^2-100^2 \rightarrow \boxed{x=\quad}$

we can find $\frac{d\alpha}{dt}$.

Apr 11-9:28 AM

Class QZ 17:

Find $\boxed{\text{all } x\text{-Values}}$ for all possible $\boxed{\text{Critical points}}$ and $\boxed{\text{inflection points}}$ for

$f(x)=x^4-2x^2-2.$

$f'(x)=4x^3-4x$ $\qquad f'(x)=0 \rightarrow 4x(x^2-1)=0$ $\qquad \boxed{x=0,\ x=\pm 1}$

$f''(x)=12x^2-4$ $\qquad f''(x)=0 \rightarrow 4(3x^2-1)=0 \rightarrow \boxed{x=\pm\frac{\sqrt{3}}{3}}$

Apr 11-9:42 AM